Relativistic Energy States of the Λ-Particle in Hypernuclei Using Gaussian Potentials

C. G. Koutroulos

Department of Theoretical Physics, University of Thessaloniki, Thessaloniki, Greece

Z. Naturforsch. **45a**, 14–16 (1990); received October 9, 1989

The Dirac equation with scalar potential and fourth component of vector potential of the Gaussian form is solved numerically for potential parameters obtained by a least squares fitting of the ground state binding energies of the Λ in a number of hypernuclei. The binding energies in the ground and excited states for various hypernuclei are determined. The spacings between the various levels are also given.

In the present work, assuming that the motion of the Λ-particle in hypernuclei is described by the Dirac equation with an average Λ-nucleus potential made up by an attractive scalar relativistic single particle potential $U_S(r)$ (generated by a scalar boson exchange interaction) and a repulsive relativistic single particle potential $U_V(r)$ which is the fourth component of a vector potential (resulting from the fourth component of a vector boson exchange interaction) or equivalently by the potentials $U_\pm(r) = U_S(r) \pm U_V(r)$ both of the Gaussian form $U_\pm(r) = -D/e^{r^2/R^2}$ with $R = r_0 A_{\text{core}}^{1/3}$ and using the parameters of these potentials (i.e. the well depths D_+, D_- and r_0) determined by a least squares fitting procedure of the ground state binding energies of the Λ in hypernuclei obtained from the second order Schrödinger like differential equation derived from the first order Dirac equation, we calculate the binding energies of the Λ-particle in various hypernuclei in the states $1s$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, $1f_{7/2}$, $1f_{5/2}$, $1g_{9/2}$, $1g_{7/2}$. We compare these binding energies with the ones obtained for the Woods-Saxon potentials [1] and also with the experimental results of Chrien [2]. Also the energy splittings Δ between the various states also given and compared with the Woods-Saxon as well as the experimental ones. The method and formalism used are outlined in [1].

Reprint requests to Dr. C. G. Koutroulos, Department of Theoretical Physics, University of Thessaloniki, Thessaloniki, Griechenland.

Table 1. Binding energies (in MeV) of the ground and excited states as well as spacings between the (averaged) states for a Λ-particle in hypernuclei and for a number of hypernuclei obtained from eq. (2) using as potential parameters for the Gaussian potentials (in a relativistic treatment) the following: $D_+ = 33.6$ MeV, $D_- = 427$ MeV and $r_0 = 1.27$ fm, $\chi^2 = 21.968$. For R the simple expression $R = r_0 A_{\text{core}}^{1/3}$ was used.

A_c	Hypernuclei	$s_{1/2}$ B_Λ	$p_{3/2}$ B_Λ	$p_{1/2}$ B_Λ	p B_Λ	p_c B_Λ	Δ_{sp}	$d_{5/2}$ B_Λ	$d_{3/2}$ B_Λ	d B_Λ	d_c B_Λ	Δ_{pd}	$f_{7/2}$ B_Λ	$f_{5/2}$ B_Λ	f B_Λ	f_c B_Λ	Δ_{df}	$g_{9/2}$ B_Λ	$g_{7/2}$ B_Λ	g B_Λ	g_c B_Λ	Δ_{fg}
8	${}^{9}_{\Lambda}$Be	8.6	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
10	${}^{11}_{\Lambda}$B	10.3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
11	${}^{12}_{\Lambda}$C	11.0	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
12	${}^{13}_{\Lambda}$C	11.6	0.3	–	–	0.1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
15	${}^{16}_{\Lambda}$O	13.2	1.7	0.7	1.2	1.3	12.0	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
27	${}^{28}_{\Lambda}$Si	17.0	6.0	5.0	5.5	5.7	11.5	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
31	${}^{32}_{\Lambda}$S	17.8	7.0	6.1	6.6	6.7	11.2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
39	${}^{40}_{\Lambda}$Ca	19.1	8.7	7.9	8.3	8.4	10.8	0.8	0.1	0.5	0.5	7.8	–	–	–	–	–	–	–	–	–	–
50	${}^{51}_{\Lambda}$V	20.5	10.5	9.8	10.2	10.3	10.3	2.7	1.8	2.3	2.3	7.9	–	–	–	–	–	–	–	–	–	–
88	${}^{89}_{\Lambda}$Y	23.2	14.4	13.8	14.1	14.2	8.9	6.9	6.2	6.6	6.6	7.5	0.9	0.2	0.6	0.6	6.0	–	–	–	–	–
137	${}^{138}_{\Lambda}$Ba	25.1	17.2	16.7	17.0	17.0	8.1	10.2	9.6	9.9	9.9	7.1	4.3	3.6	4.0	4.1	5.9	–	–	–	–	–
207	${}^{208}_{\Lambda}$Pb	26.7	19.5	19.1	19.3	19.3	7.4	13.1	12.6	12.9	12.9	6.4	7.4	6.8	7.1	7.1	5.8	2.5	2.0	2.3	2.3	4.8

0932-0784 / 90 / 0100-0014 $ 01.30/0. – Please order a reprint rather than making your own copy.

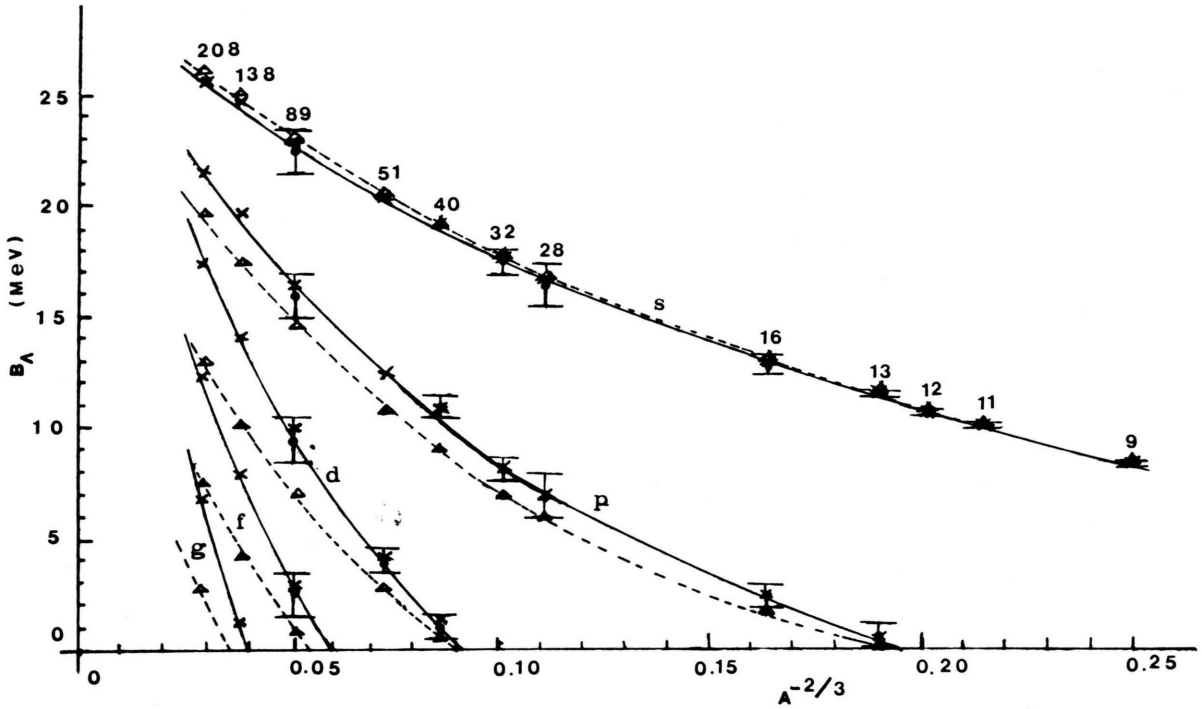

Fig. 1. The binding energies of the Λ-particle in the states 1s, 1p, 1d, 1f, and 1g.

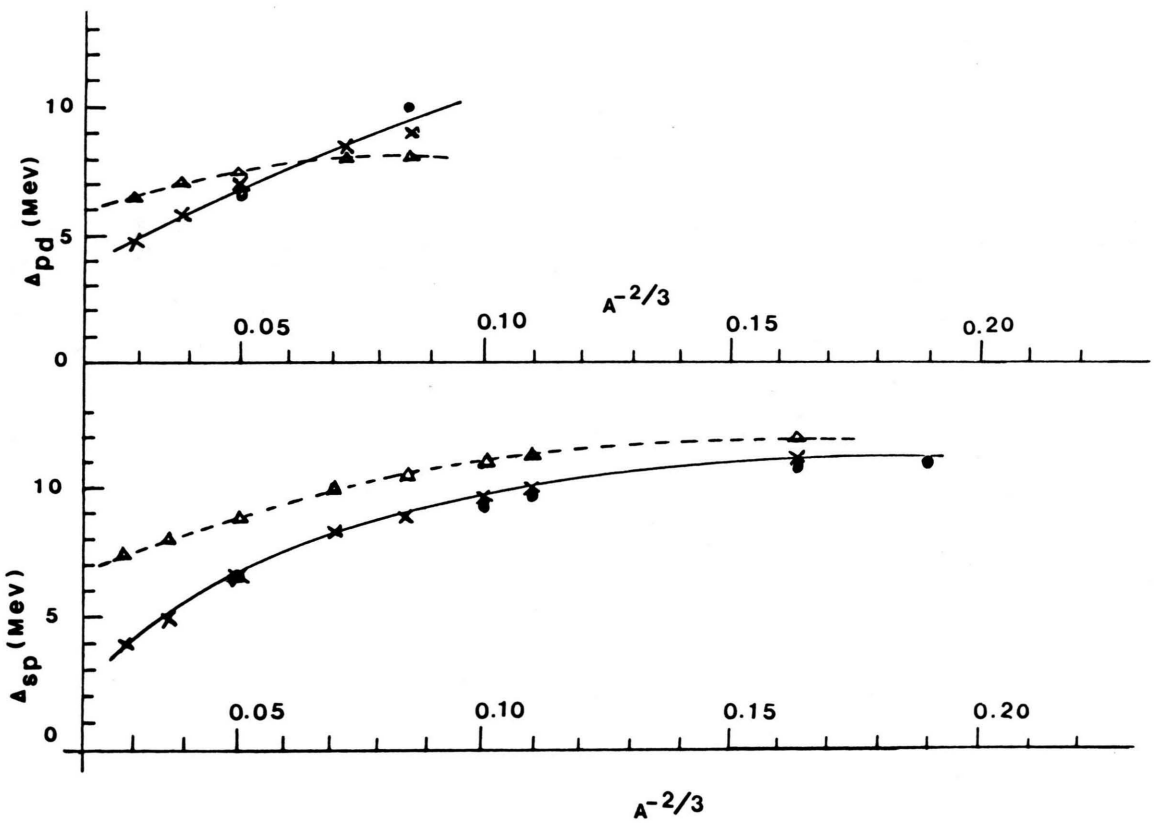

Fig. 2. The splittings Δ_{sp} and Δ_{pd} of various hypernuclei.

The best fit values of the potential parameters obtaind for the Gaussian potentials are

$$D_+ = 33.6 \text{ MeV}, \quad D_- = 427.0 \text{ MeV},$$

$$r_0 = 1.27 \text{ fm} \quad (\text{with } \chi^2 = 21.968).$$

Because the experimental resolution is around 3 MeV and the splitting of the levels due to the spin orbit coupling is much less, we also give the binding energies of the unsplitted states 1 s, 1 p, 1 d, 1 f, 1 g, resulting by taking the average of the binding energies of the corresponding splitted ones. Also we give the binding energies of the states $1s_c$, $1p_c$, $1d_c$, $1f_c$, $1g_c$ resulting using in the Schrödinger like equation only the central part of the potential $V_{\text{centr.}}$, i.e. keeping the spin orbit part of the potential (i.e. $V_{\text{S.O.}}$) equal to zero. This shows that $V_{\text{S.O.}}$ affects the states very little.

Our results are shown in Table 1. Figure 1 shows how well the binding energies of the Λ-particle in the states 1 s, 1 p, 1 d, 1 f, 1 g given in Table 1 fit the experimental results of Chrien [2]. Also in the same figure the results obtained Woods-Saxon potentials with parameters $D_+ = 30$ MeV, $D_- = 443$ MeV, $r_0 = 1.21$ fm, $\alpha = 0.6$ fm are given [1]. The triangles and dashed curves correspond to the Gaussian case, the crosses and continuous curves to the Woods-Saxon case. The experimental points are indicated by dots. In Fig. 2 the splittings Δ_{sp} and Δ_{pd} of various hypernuclei are given. The description of the points and curves is the same as above.

We notice that in the ground state the binding energies obtained using the Gaussian potentials as well as those obtained using the Woods-Saxon potentials are in good agreement with the experimental results of Chrien [2]. In the excited states the binding energies obtained using the Gaussian potentials are not in so good agreement with the experimental ones while the Woods-Saxon ones are. We have the feeling that the results for the Gaussian case in the excited states can be improved either by using a more complicated expression for the potential radius or by performing the least squares fitting using the binding energies of the excited states.

Thanks are due to Prof. M. Grypeos, Associate Prof. S. Massen and Dr. G. Lalazissis.

[1] C. G. Koutroulos, Z. Naturforsch. **44a**, 1234 (1989).
[2] R. E. Chrien, Proc. of the XI[th] International Conference on Particles and Nuclei, Kyoto, April 1987, Nucl. Phys. **A 478**, 705c (1988).